Name _____

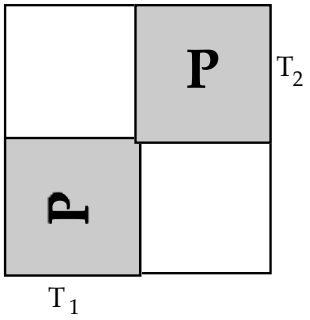
Linear Algebra/Differential Equations
Practice Midterm Exam

Getting ready for April 3, 2009

with Dr. Crannell

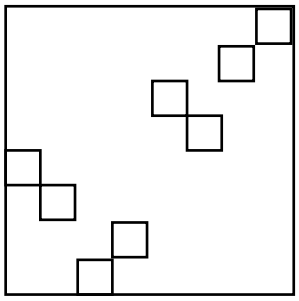
problem	points	your score
1	10	_____
2	10	_____
3	20	_____
4	20	_____
5	20	_____
6	20	_____
Total	**100**	_____

1. Consider the transformations drawn below. (The big square represents the initial picture, and the two shaded regions represent the two transformations).



(a) Draw the second iterate (include 'P's).

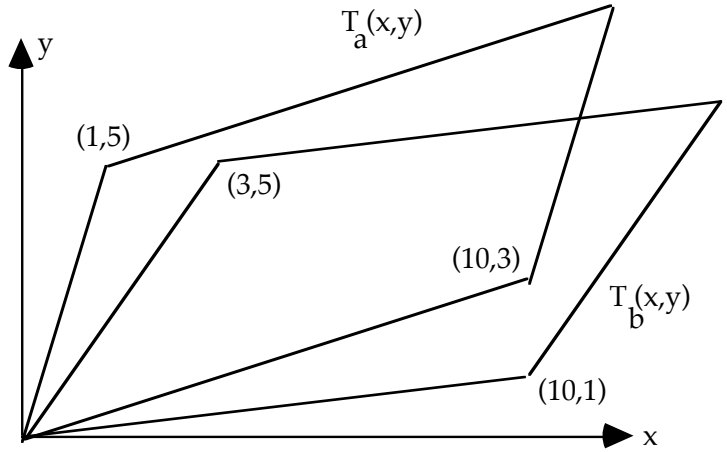
(b) Pictured here is the third iterate. Identify $T_1 \circ T_2 \circ T_1(I^2)$ with a (*) and $T_2 \circ T_2 \circ T_2(I^2)$ with a ($).



2. Compute the eigenvalues and corresponding eigenvectors of the matrix $\begin{pmatrix} 16 & -10 \\ 18 & -11 \end{pmatrix}$

3. Below are the images of two affine transformations applied to the unit square. One of these transformations is $T_1\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 & 3 \\ 1 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$, and the other is $T_2\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 & 1 \\ 3 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$.

(a) Which is which?



a = _____

b = _____

(b) Compute the algebraic formula for $T_1 \circ T_2\begin{pmatrix} x \\ y \end{pmatrix}$.

(c) Draw the image $T_1 \circ T_2\left(I^2\right)$.

4. You know (because I am your professor, and I'm telling you it's true) that T:$\mathbf{R}^3$ —>$\mathbf{R}^3$ is a linear transformation.

a) If $T\begin{pmatrix}1\\0\\1\end{pmatrix}=\begin{pmatrix}2\\0\\2\end{pmatrix}$ and $T\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{pmatrix}2\\2\\0\end{pmatrix}$, either determine $T\begin{pmatrix}0\\1\\1\end{pmatrix}$ or explain why it impossible to do so.

b) If $T\begin{pmatrix}1\\0\\1\end{pmatrix}=\begin{pmatrix}2\\0\\2\end{pmatrix}$, $T\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{pmatrix}2\\2\\0\end{pmatrix}$, and $T\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}1/2\\0\\5/2\end{pmatrix}$ either determine $T\begin{pmatrix}0\\1\\1\end{pmatrix}$ or explain why it impossible to do so.

5. (a) Determine the general solution to $\frac{dy}{dx} + 2xy = 0$.

(b) Determine any particular solution to $\frac{dy}{dx} + 2xy = 2x$.

c) Is the differential equation in part b)

. . . homogeneous?
. . . separable?
. . . exact?
. . . linear?
. . . .

6. The function $\mathbf{x}(t) = \begin{pmatrix} te^t - e^t \\ e^t \end{pmatrix}$ is a solution to the differential equation $D\mathbf{x} = N\mathbf{x}$.

a) Find the matrix N.

b) Find at least one *other* solution to the differential equation.

c) Sketch the solutions to the differential equation.